K26FY1590 Pages: 2 Reg No:.....
Name:.....

Second Semester FYUGP Degree (Reg/Sup/Imp) Examination April 2026

KU2DSCMAT116 - MULTIVARIABLE CALCULUS

2024 Admission onwards

Time : 2 hours Maximum Marks : 70

Section A

Answer any 6 questions. Each carry 3 marks.

1. Evaluate $\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$.
2. Sketch the region $0 \leq x \leq 2, 0 \leq y \leq 2x$.
3. Give normal vector of a surface $f(x, y, z) = c$ at the point (a, b, c), where f is a differentiable scalar function.
4. Define divergence of a vector field.
5. Find the divergence of the vector field $[3xz, 2xy, -yz^2]$ at the point (1,2,-3) .
6. Show that $\mathbf{F} = (2x - 3)\mathbf{i} - z\mathbf{j} + (\cos z)\mathbf{k}$ is not conservative.
7. Find the work done by the force field $F = \nabla f$, where $f(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$ in moving an object along a smooth curve C joining $(1, 0, 0)$ to $(0, 0, 2)$ that does not pass through the origin.
8. State the formula to find the mass of a thin shell of density $\delta(x, y, z)$.

Section B

Answer any 4 questions. Each carry 6 marks.

9. Define gradient of a scalar differentiable function $f(x, y, z)$. Using the idea of gradient, find the unit surface normal of the surface $z^2 = 4x^2 + y^2$ at the point $(1,1,\sqrt{5})$.
10. If $f(x, y, z)$ is a scalar function, prove that $\nabla(f^n) = nf^{n-1}\nabla f$.
11. Find the curl of the vector function $[xy, y^2z, xyz]$ at the points (1,2,1) and (1,0,-4).
12. Evaluate $\int_C \sqrt{(x^2 + y^2)}\, ds$ along the curve $\mathbf{r}(t) = (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j} + 3t\mathbf{k},\ -2\pi \leq t \leq 2\pi$.
13. Find the parametrization of the paraboloid $z = x^2 + y^2,\ z \leq 4$.

14. Find a parametrization of the cylinder $x^2 + (y-3)^2 = 9, \quad 0 \le z \le 5$.

Section C

Answer any 2 questions. Each carry 14 marks.

15. (a) Let $f(x,y) = \begin{cases} x^2, & x \ge 0 \\ x^3, & x < 0 \end{cases}$.

Find the following limits.

(i) $\lim\limits_{(x,y)\to(3,-2)} f(x,y)$

(ii) $\lim\limits_{(x,y)\to(-2,1)} f(x,y)$.

(b) If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an $R-$ohm resistor, the value of R can be found from the equation $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$. Find the value of $\dfrac{\partial R}{\partial R_2}$ when $R_1 = 30, R_2 = 45$ and $R_3 = 90$ ohms.

16. (a) Use the chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t,\ y = \sin t$. Find the value of $\dfrac{dw}{dt}$ at $t = \dfrac{\pi}{2}$.

(b) Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ if $z = \sin xy + x \sin y, x = u^2 + v^2, y = uv$.

(c) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(0,0,0)$ for $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

17. (a) Find the volume of the region bounded above by the plane $z = y/2$ and below by the rectangle $R : 0 \le x \le 4,\ 0 \le y \le 2$.

(b) Evaluate $\displaystyle\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy$.

CENTRAL LIBRARY